Coherent Exclusive Exponentiation for Precision Monte Carlo Calculations*

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In this contribution we give a short overview of the new Coherent Exclusive Exponentiation (CEEX) which is implemented in the new $\mathcal{KK}$MC event generator for the process $e^+e^- \to f\bar{f} + n\gamma$, $f = \mu, \tau, d, u, s, c, b$ with validity for center of mass energies from τ lepton threshold to 1TeV, that is for LEP1, LEP2, SLC, future Linear Colliders, b, c, τ-factories etc. In CEEX effects due to photon emission from initial beams and outgoing fermions are calculated in QED up to second-order, including all interference effects. Electroweak corrections are included in first-order, at the amplitude level. Beams can be polarized longitudinally and transversely, and all spin correlations are incorporated in an exact manner. We describe briefly the essence of CEEX as compared with older exclusive variants of the exponentiation (EEX) and present samples of the numerical results, concentrating on the question of the technical and physical precision for the total cross section and for the charge asymmetry. New results on the interference between initial and final state photon emission at LEP2 energies are also shown.

1. Introduction

At the end of LEP2 operation the total cross section for the process $e^-e^+ \to f\bar{f} + n\gamma$ will have to be calculated with the precision $0.2\% - 1\%$, depending on event selection. The arbitrary differential distributions have to be calculated also with the corresponding precision. In future linear colliders (LC's) the precision requirement can be even more demanding. This is especially true for high luminosity linear colliders, like in the case of TESLA. The above new requirements necessitate development of the new calculational framework for the QED corrections and the construction of new dedicated MC programs. We present here a new effort in this direction. This report is based on refs. [1–3] and the Monte Carlo program is described in ref. [4].

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2. What is coherent exclusive exponentiation CEEX?

The *exponentiation* is generally a method of summing up real and virtual photon contributions to infinite order such that infrared (IR) divergences cancel. The *exclusivity* means that the procedure of exponentiation, that is summing up the infrared (IR) real and virtual contribution, within the standard perturbative scheme of quantum field theory, is done at the level of the fully differential (multiphoton) cross section, or even better, at the level of the scattering matrix element (spin amplitude), *before any phase-space integration over photon momenta is done*. The other popular type of the exponentiation is *inclusive* exponentiation (IEX), which is done at the level of inclusive distributions, structure functions, etc. see discussion in ref. [5]. The classi-

Feature	KORALB	KORALZ	$\mathcal{KK}$ 4.13	$\mathcal{KK}$ 2000+?
QED type	$\mathcal{O}(\alpha)$	EEX	CEEX, EEX	CEEX, EEX
CEEX(ISR+FSR)	none	none	$\{\alpha, \alpha L; \alpha^2 L^2, \alpha^2 L^1\}$	$\{...\alpha^2 L^1; \alpha^3 L^3\}$
EEX(ISR*FSR)	none	$\{\alpha, \alpha L, \alpha^2 L^2\}$	$\{\alpha, \alpha L, \alpha^2 L^2, \alpha^3 L^3\}$	$\{...\alpha^2 L^2, \alpha^3 L^3\}$
ISR-FSR int.	$\mathcal{O}(\alpha)$	$\mathcal{O}(\alpha)$	$\{\alpha, \alpha L\}_{\rm CEEX}$	$\{\alpha, \alpha L\}_{\rm CEEX}$
Exact bremss.	1 γ	1, 2coll. γ	1, 2, 3coll. γ	up to 3 γ
Electroweak	No Z-res.	DIZET 6.x	DIZET 6.x	New version?
Beam polar.	long+trans.	longit.	long+trans.	long+trans.
τ polar.	long+trans.	longit.	long+trans.	long+trans.
Hadronization	—	JETSET	JETSET	PYTHIA
τ decay	TAUOLA	TAUOLA	TAUOLA	TAUOLA
Inclusive mode	—	No	Yes	Yes
Beamstrahlung	—	No	Yes	Yes
Beam spread	—	No	Yes	Yes
$\nu\nu$ channel	—	Yes	No	Yes
ee channel	—	No	No	Yes
tt channel	—	No	No	yes?
WW channel	—	No	No	yes?

Table 1
Overview of $\mathcal{KK}$MC event generator as compared with KORALZ and KORALB.

cal work of Yennie-Frautschi-Suura [6] (YFS) represents the best example of the exclusive exponentiation and we nickname it as EEX. Finally, why do we use word *coherent?* In CEEX the essential part of the summation of the IR real and virtual photon contributions is done at the amplitude level. Of course, IR cancellations occur as usual at the probability level, however, the transition from spin amplitudes to differential cross sections, and the phase space integration are done entirely numerically! As a consequence of the above *coherent* approach it follows, that CEEX is friendly to coherence among Feynman diagrams, narrow resonances, interferences etc. This is great practical advantage. In our many previous works which led to the development of the Monte Carlo event generators like YFS3, YFS3, KORALZ, KORALW, YFS3WW, BHLUMI, BHWIDE, see refs. [7–12], we have generally employed EEX, which is closely related to the YFS work [5]. The CEEX is a recent development and is so far used only in the new $\mathcal{KK}$MC program [4].

Let us now show in a very simplified schematic way what is the the main difference between old EEX/YFS and CEEX for the fermion pair production the process:

$$e^-(p_1,\lambda_1) + e^+(p_2,\lambda_2) \to$$
$$f(q_1,\lambda'_1) + \bar{f}(q_2,\lambda'_2) + \gamma(k_1,\sigma_1) + ... + \gamma(k_n,\sigma_n). \quad (1)$$

The EEX total cross section is

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2}\, e^{Y(m_\gamma)} D_n(q_1,q_2,k_1,...,k_n), \quad (2)$$

where in the $\mathcal{O}(\alpha^1)$ the distributions for $n_\gamma = 0,1,2$ are

$$\begin{aligned} D_0 &= \bar\beta_0 \\ D_1(k_1) &= \bar\beta_0 \tilde{S}(k_1) + \bar\beta_1(k_1) \\ D_2(k_1,k_2) &= \bar\beta_0 \tilde{S}(k_1)\tilde{S}(k_2) \\ &\quad + \bar\beta_1(k_1)\tilde{S}(k_2) + \bar\beta_1(k_2)\tilde{S}(k_1) \end{aligned} \quad (3)$$

and the real soft factors are defined as usual

$$4\pi\tilde{S}(k) = \sum_{\sigma} |\mathfrak{s}_{\sigma}(k)|^2 = |\mathfrak{s}_{+}|^2(k) + |\mathfrak{s}_{-}(k)|^2 \\ = -\alpha\pi\left(q_1 k q_1 - q_2 k q_2\right)^2. \quad (4)$$

What is important for our discussion is that the IR-finite building blocks

$$\bar{\beta}_0 = \sum_{\lambda} |\mathcal{M}_{\lambda}|^2, \\ \bar{\beta}_1(k) = \sum_{\lambda\sigma} |\mathcal{M}^{1-\mathrm{phot}}_{\lambda\sigma}|^2 - \sum_{\sigma} |\mathfrak{s}_{\sigma}(k)|^2 \sum_{\lambda} |\mathcal{M}^{\mathrm{Born}}_{\lambda}|^2 \quad (5)$$

in the multiphoton distributions are all in terms of $\sum_{spin} |...|^2$!! We denoted: λ = fermion helicities and σ = photon helicity.

The above is to be contrasted with the analogous $\mathcal{O}(\alpha^1)$ case of CEEX

$$\sigma = \sum_{n=0}^{\infty} \int_{m_\gamma} d\Phi_{n+2} \\ \sum_{\lambda,\sigma_1,...,\sigma_n} |e^{B(m_\gamma)} \mathcal{M}^{\lambda}_{n,\sigma_1,...,\sigma_n}(k_1,...,k_n)|^2, \quad (6)$$

where the differential distributions for $n_\gamma = 0, 1, 2$ photons are the following:

$$\mathcal{M}^{\lambda}_0 = \hat{\beta}^{\lambda}_0, \quad \lambda = \mathrm{fermionhelicities}, \\ \mathcal{M}^{\lambda}_{1,\sigma_1}(k_1) = \hat{\beta}^{\lambda}_0 \mathfrak{s}_{\sigma_1}(k_1) + \hat{\beta}^{\lambda}_{1,\sigma_1}(k_1), \\ \mathcal{M}^{\lambda}_{2,\sigma_1,\sigma_2}(k_1,k_2) = \hat{\beta}^{\lambda}_0 \mathfrak{s}_{\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) \\ + \hat{\beta}^{\lambda}_{1,\sigma_1}(k_1)\mathfrak{s}_{\sigma_2}(k_2) + \hat{\beta}^{\lambda}_{1,\sigma_2}(k_2)\mathfrak{s}_{\sigma_1}(k_1) \quad (7)$$

and the IR-finite building blocks are

$$\hat{\beta}^{\lambda}_0 = \left(e^{-B}\mathcal{M}^{\mathrm{Born+Virt.}}_{\lambda}\right)\big|_{\mathcal{O}(\alpha^1)}, \\ \hat{\beta}^{\lambda}_{1,\sigma}(k) = \mathcal{M}^{\lambda}_{1,\sigma}(k) - \hat{\beta}^{\lambda}_0 \mathfrak{s}_{\sigma}(k). \quad (8)$$

As shown explicitly, this time everything is in terms of $\mathcal{M}$-spin-amplitudes! This is the basic difference between EEX/YFS and CEEX. The complete expressions for spin amplitudes with CEEX exponentiation, for any number of photons, are shown in ref. [1] for the $\mathcal{O}(\alpha^1)$ case and in ref. [3] for the $\mathcal{O}(\alpha^2)$ case.

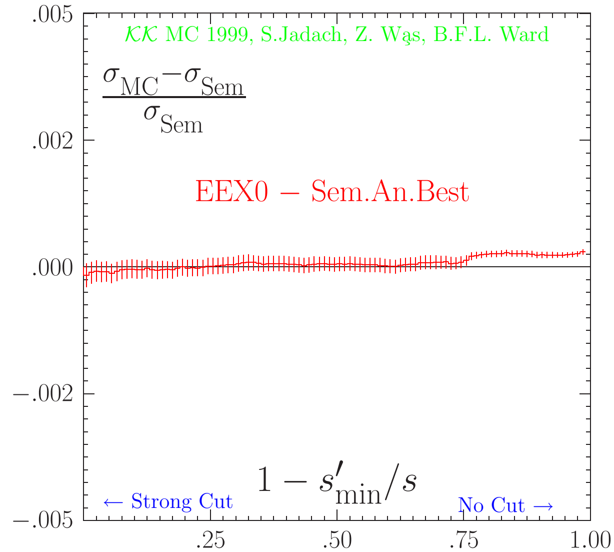

Figure 1. Baseline test of the technical precision.

3. Monte Carlo numerical results

The $\mathcal{O}(\alpha^2)$ CEEX-style matrix element is implemented in $\mathcal{KK}$MC which simulates production of muon, tau and quark pairs. Electrons (Bhabha scattering) and neutrino channels are not available. The program includes for the optional use the older, EEX-style matrix element. It is functionally similar to KORALZ [8] and the older KORALB [13] programs. In Table 1 we provide the complete comparison of features of $\mathcal{KK}$MC and the older programs.

3.1. Technical precision

For the new MC program of the high complexity like $\mathcal{KK}$MC it is important to check very precisely the overall normalization. This is the cornerstone of the evaluation of the *technical precision* of the program, especially for $\mathcal{KK}$MC which is aimed at the end of testing at the total precision of 0.1%. In Fig. 1 we present the comparison of the $\mathcal{KK}$MC with simple semianalytical integration for the total cross section, as a function of the minimum mass $\sqrt{s'_{\mathrm{min}}}$ of the final muon pair. It is

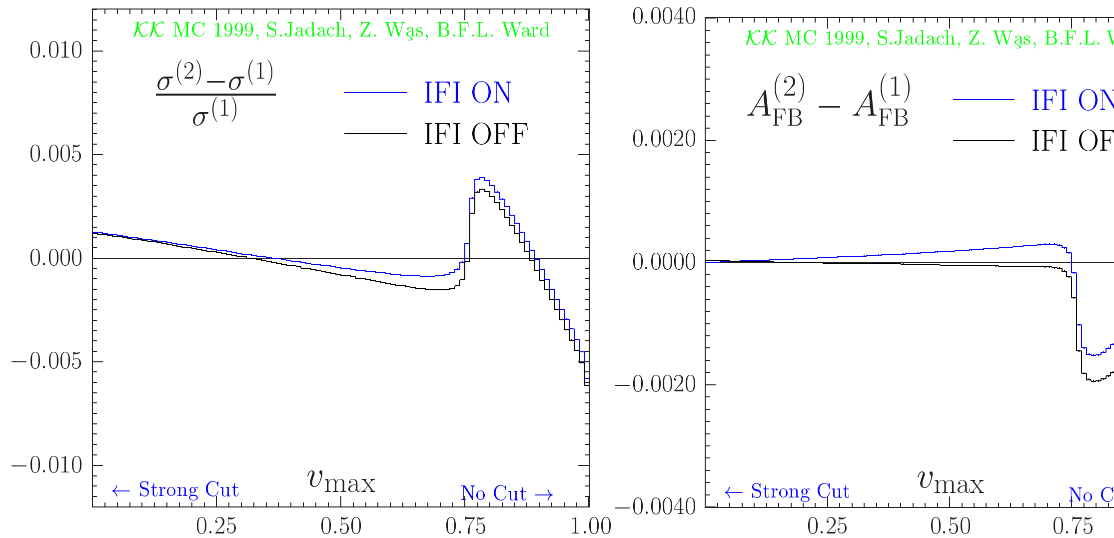

Figure 2. Test of the technical precision of $\mathcal{KK}$MC.

done for muon-pair final state at $\sqrt{s} = 200GeV$. For $\sqrt{s'_{\rm min}} \to \sqrt{s}$, when emission of hard photons is suppressed, there is an agreement $< 0.02\%$ between $\mathcal{KK}$MC and the analytical calculation. For $\sqrt{s'_{\rm min}} < M_Z$ the on-shell Z-boson production due to emission of the hard initial state radiation (ISR), the so called Z radiative return (ZRR), is allowed kinematically. Even in this case (more sensible to higher orders) the agreement $< 0.02\%$ is reached. For the above exercise we used the simplified $\mathcal{O}(\alpha^0)$ CEEX matrix element, because in this case the precise phase analytical integration is relatively easy.

3.2. Physical precision

The equally important component of the overall error is the physical error which we estimate conservatively as the half of the difference $\mathcal{O}(\alpha^2){-}\mathcal{O}(\alpha^1)$. In Fig. 2 we show the corresponding result for the total cross section and charge asymmetry for $\sqrt{s} = 189GeV$ as a function of the cut on energies of all photons ($s'_{\rm min} > s$ limits the total photon energy.) We obtain in this way the estimate 0.2% for the physical precision of the total cross section and 0.1% for the charge asymmetry. Both plots in Fig. 2 show as expected strong variation at the position of the ZRR. This precision is good enough for the LEP2 combined data.

3.3. Initial-final state QED interference

One important benefit from CEEX with respect to the older EEX is the inclusion of the Initial-Final state QED Interference (IFI). The effect of IFI is comparable with the precision of the LEP2 combined data and should be under good control. Results of our analysis of the size of IFI at LEP2 energies ($\sqrt{s} = 189GeV$) are shown in Fig. 3. In this figure we compare the CEEX result of $\mathcal{KK}$MC first of all with the result of KORALZ which is run in the $\mathcal{O}(\alpha^1)$ mode without exponentiation (IFI is neglected for KORALZ with the EEX matrix element.) The $\mathcal{O}(\alpha^1)$ IFI contribution from KORALZ was extensively cross-checked in the past with the dedicated semi-analytical calculations [14], it is therefore a good reference and starting point. As we see the IFI contribution of CEEX differs slightly from the pure $\mathcal{O}(\alpha^1)$ result. It is related to exponentiation which makes the angular dependence (in the muon scattering

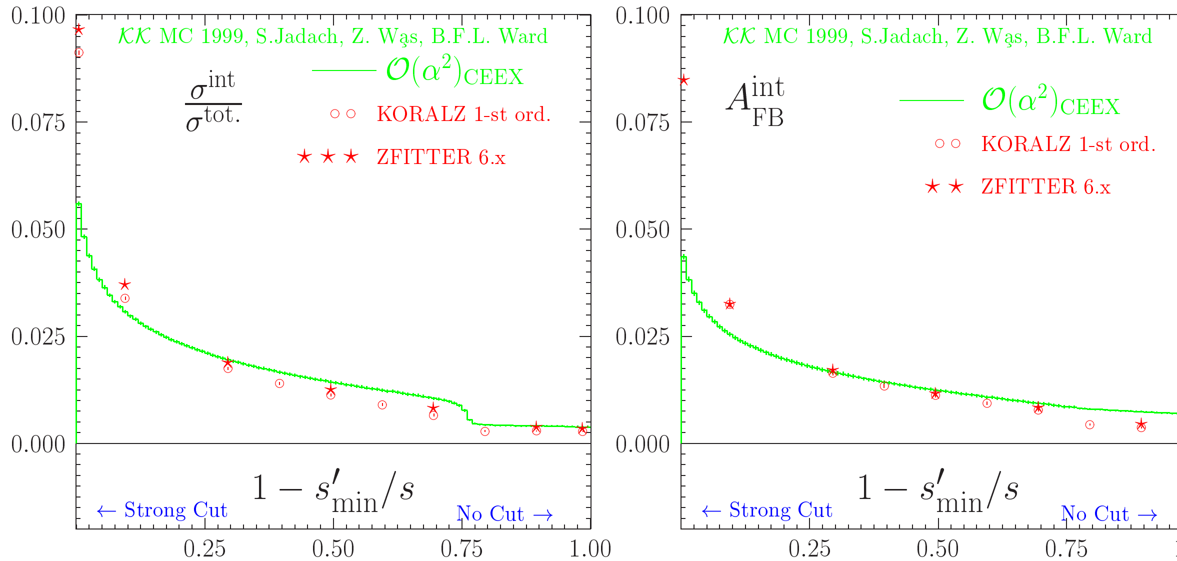

Figure 3. The effect of the initial-final state QED interference in total cross-section and charge asymmetry.

angle) of the IFI contribution less sharp and it is also due to convolution of IFI with the $\mathcal{O}(\alpha^2)$ ISR. The expected modification of the interference correction due to higher orders is about 20% for cross section and asymmetry, if ZRR is excluded, (the size of ISR correction in the cross section) and it is indeed of this size. Apparently, this principle works also in the case of ZRR included, remembering that in this case ISR correction is 100% or more. However, we feel that this case requires further study. We have also included results of the semianalytical program ZFITTER [15] in our plots[2]. They agree well with the $\mathcal{O}(\alpha^1)$ IFI of KORALZ. This is expected because they are without exponentiation.

4. Outlook and summary

The most important new features in the present CEEX are the ISR-FSR interference, the second-order subleading corrections, and the exact matrix element for two hard photons. This makes CEEX already a unique source of SM predictions for the LEP2 physics program and for the LC physics program. Note that for these the electroweak correction library has to be reexamined at LC energies. The most important omission in the present version is the lack of neutrino and electron channels. Let us stress that the present program is an excellent starting platform for the construction of the second-order Bhabha MC generator based on CEEX exponentiation. We hope to be able to include the Bhabha and neutrino channels soon, possibly in the next version. The other important directions for the development are the inclusion of the exact matrix element for three hard photons, together with virtual corrections up to $\mathcal{O}(\alpha^3 L^3)$ and the emission of the light fermion pairs. The inclusion of the W^+W^- and $t\bar{t}$ final states is still in a farther perspective.

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